Review of Seats

Problem

The numbers between 0 and $HW-1$, inclusive, are written on a rectangle divided to $H \times W$ squares such that each square contains one number and each number is written on exactly one square.

Deal with the following queries one by one:

- Swap the positions of two numbers.
- Count the number of t between 0 and $HW-1$ such that the squares where the numbers between 0 and t are written form a rectangle.

Subtasks and Solutions

Subtask 1 (HW is very small)

Determine whether a set of squares forms a rectangule or not in $O(HW)$ time and deal with each query in $O(H^2W^2)$ time.

Subtask 2 (HW is small)

Let the square where the number i is written be (r_i, c_i). The sufficient and necessary condition of $(r_0, c_0), \ldots (r_t, c_t)$ forming a rectangle is

$$(\max r_i - \min r_i + 1)(\max c_i - \min c_i + 1) = t + 1.$$

Here i moves between 0 and t.

Check if these conditions hold from $t = 0$ to $t = HW-1$ inductively in $O(HW)$ time.

Subtask 3 ($H + W$ is small)

If $(\max r_i - \min r_i + 1)(\max c_i - \min c_i + 1) > t + 1$, the next t satisfying the condition is not less than $(\max r_i - \min r_i + 1)(\max c_i - \min c_i + 1) - 1$.

So it is enough to determine this condition in $O(H + W)$ numbers.

This condition is determinable independently in $O(\log(HW))$ time using a segment tree. Then each query is dealt with in $O((H+W)\log(HW))$ time.

Subtask 4 (Gaps of two swapped numbers are small)

Memorize max and min of r_i and c_i for each t and recalculate them for t between the two swapped numbers for each query.

Subtask 5 (One dimensional, namely, $H = 1$)

Take t arbitrarily and color the squares with numbers $0, \ldots, t$ black and the squares with numbers $t+1, \ldots, W-1$ white (and add white squares outside of the rectangle.)

The sufficient and necessary condition of black squares forming a rectangle is that the number of pairs of adjacent two squares with different colors is two.

Manage the numbers of such pairs of squares for all values of t and count the number of ts satisying the condition efficiently using a lazy propagation segment tree and deal with each query in $O(\log W)$ time.

Subtask 6 (No additional constraints)

Like above, take t arbitrarily and color the squares with numbers $0, \ldots, t$ black the squares with numbers $t+1, \ldots, HW-1$ white (and add white squares outside of the rectangle.)

Pay attention to two-times-two squares (sets of adjacent four squares.)

The sufficient and necessary condition of black squares forming a rectangle is as follows:

- There are only four two-times-two squares which contain exactly one black square.
- There are no two-times-two squares which contain exactly three black squares.

Manage the numbers of such two-times-two squares for all values of t and count the number of ts satisying the condition efficiently using a lazy propagation segment tree and deal with each query in $O(\log(HW))$ time.